21. Find all the cluster points of $\cos\frac{1}{x}$ at $x=0$.

22. Show that $\lim_{n\to\infty}\left(\sum_{k=1}^{n}\frac{1}{k!}\right)$ exists.

23. Show that $\sum_{k=1}^{n}\frac{1}{k}$ diverges.

24. Show that if $\sum_{n=1}^{\infty}a_n$ converges then $\lim_{n\to\infty}a_n=0$.

25. Show that f is a constant if $|f(x)-f(y)|<2(x-y)^2$ for all x and y.

26. Show that $|\sin x-\sin y|\le|x-y|$ for all x and y.

27. Suppose that a sequence $\{x_n\}$ has the property that there is an r, $(0<r<1)$, for which $|x_{n+1}-x_n|<cr^n$.
Show that $\lim_{n\to\infty}x_n$ exists.

28. Compute the following limits.

a) $\lim_{x\to\infty}x^{\frac{1}{x}}$ b) $\lim_{x\to 0}\frac{a^x-b^x}{\sin x}$, $a>0,\ b>0$

29. Evaluate $\lim_{n\to\infty}\sum_{k=1}^{n}\frac{k}{n^2+k^2}$.

30. Show that if f is integrable on $I:\{a\le x\le b\}$ then so is $|f|$ and

$$\left|\int_a^b f(x)\,dx\right|\le\int_a^b|f(x)|\,dx.$$

31. Prove that $f(x_n)\to f(a)$ if $f(x)$ is continuous at $x=a$, and $x_n\to a$.

32.Show that a sequence $\{a_n\}$ converges if for each $\epsilon>0$ there is an $N(\epsilon)$ for which $|a_m-a_n|<\epsilon$ for all $n>N$, $m>N$.

33. Let f be strictly increasing and continuous on an interval I. Its range will be an interval J. Prove that the inverse function of f is strictly increasing on an interval J.

34 Show that for each nonnegative number y, there is a unique $x \geq 0$ for which $x^2 = y$.

35. Prove that if f is monotone on an interval I, then f is integrable there.

36. Prove that if f is integrable on an interval I, then so is f^+.

37. If f is integrable on an interval $I : \{a \leq x \leq b\}$, show that $\int_a^x f(t)\, dt$ is continuous on I.

38. Write the first three nonzero terms in Taylor's formula for $\sin x$ at $x = \frac{\pi}{4}$.

39. Let $f(x)$ be defined for all real numbers and continuous at $x = 0$.
Suppose that $f(a+b) = f(a) + f(b)$.
Show that f is uniformly continuous everywhere.

40. Prove that a continuous function defined on a closed bounded interval is uniformly continuous there.