1. Show that if $|a_n| \to 0$ if and only if $a_n \to 0$.

2. If $\lim_{x\to a} f(x) = A$, then there is a deleted neighborhood of a where $f(x)$ is bounded.

3. Let $\{A_n\}$ be a nonincreasing sequence bounded below. Prove that there exists its limit.

4. Prove that $g \circ f$ is continuous if $f, g : \mathbb{R} \to \mathbb{R}$ are continous.

5. Prove that a bounded infinite set has at least one accumulation point.

6. Show that

$$\limsup(a_n + b_n) \leq \limsup a_n + \limsup b_n$$

7. Prove that if f is continuous on a closed bounded interval, then it is bounded there.

8. Prove that a continuous function defined on a closed bounded interval has a maximum and minimum value.

9. Prove that if f is defined in an interval and attains a maximum value at an interior point a and if $f'(a)$ exists, then $f'(a) = 0$.

10. Show that $3x^4 + 4x^3 + c$ has at most one root less than or equal to -1 no matter what the value of c.

11. Write the first three nonzero terms in Taylor's formula for e^{x^2} at $x = 0$.

12. Write the first three nonzero terms in Taylor's formula for $\arcsin x$ at $x = 0$.

13. Prove that if $a_n \geq 0$ and $b_n \geq 0$, then $\limsup(a_n b_n) \leq [\limsup a_n][\limsup b_n]$

14. Prove that $f(x) = 1/x$ is not uniformly continuous in $(0, 1]$.

15. Let $f(x)$ be a continuous function on an interval I satisfying that $f(x) = 0$ whenever x is irrational. Show that $f(x) \equiv 0$ on I.

16. Prove that if f is differentiable on an interval I and if $f'(x) \geq 0$ there, then f is nondecreasing.

17. Define $f(x) = x^2$ for rational x and $f(x) = 0$ otherwise. Prove that f is differentiable at only one point.

18. Prove that if the limit of $\{a_n\}$ exists, then $\{a_n\}$ is bounded.

19. Show that $n^n/n!e^n$ tends to a limit.

20. Prove that $f(x) = \sin x$ is uniformly continuous for all x.